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Abstract

We showed that in Lamb-Dicke regime and under rotating wave approximation, the dynamical behavior of two trapped ions interacting with a laser beam resonant to the first red side-band of center-of-mass mode can be described by Jaynes-Cummings Model. An exact analytic solution for this kind of Jaynes-Cummings model is presented. The results showed that quantum collapses and revivals for the occupation of two atoms, and squeezing for vibratic motion of center-of-mass mode existed in both two different types of initial conditions. The maximum momentum squeezing for center-of-mass mode in these two types of conditions are found to be 42.4% and 43.8% respectively. The coherence, in the first type of initial conditions can keeps long times, and in the second type of initial conditions, a concrete form of coherent state is obtained, when the initial average number is very small.

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I. INTRODUCTION

Jaynes-Cummings model(JCM) [1] originating from quantum optics that describes the idealized situation of resonant interaction of an undamped two-level system with a single, quantized harmonic oscillator turned out to be one of the fundamental models in NMR, quantum optics, quantum electronics and resonance physics. This model can be exactly solved and predicts some interesting nonclassical effects both with respect to the atomic states and the boson mode, such as atomic revivals[2] and squeezing[3]. So this model has been studied extensively in both theoretical and experimental field.

In the field of cavity QED, experimental realization of JCM with dissipation has been demonstrated both in microwave and in optical domain. Observation of quantum revivals and nonclassical photon statistics with Rydberg atoms and microwave cavities has been reported [4], and in the optical regime vacuum Rabi splitting has been observed [5]. These experimental progress stimulated a manifold of further theoretical studies, for example of the quantum correlation between two interacting subsystems [6]. Moreover, related models have been rendered successively, such as nonlinearly coupled [7] and multiquantum JCMs [8], Raman-type [9], and three-level models [10].

Another fundamental system to exhibit, in appropriate limits, JCM dynamics is a trapped ion interacting with laser beam. The quantized center-of-mass motion of the ion in the trap potential plays the role of the boson mode, which is coupled via the laser to the internal degrees of freedom. An analogy between an undamped trapped ion and the JCM has been noted by Blockley, Wall, and Risken [11], and by J. I. Cirac, et al., for damped trapped ion [12]. A study for nonlinear multiquantum JCM dynamics of a trapped ion is presented by W. Vogel and R. L. de Matos Filho [13].

In recent experiments of trapped ions, entangled states up to four ions have been realized, and some applications, such as the proof of Bell's inequality and decoherence-free quantum memory, have been confirmed[14]. These improvements offered some advantageous conditions for studying properties of multi-ion JCM. In this paper we will demonstrate that

a system comprising two trapped ions interacting with laser beam resonant to the first red side-band of center-of-mass mode, in Lamb-Dicke regime, shows a JCM-like dynamics. Collapses and revivals of the occupation of internal states, and squeezing of the vibratic motion of center-of-mass mode, can be observed in two different types of initial condition. The maximum momentum squeezing in these two different types of initial condition are 42.4% and 43.8% respectively. Coherence of center-of-mass mode is also discussed in these two types of condition. In the first type of condition, e.g. the internal state of the two atoms being in ground and the vibratic motion of center-of-mass mode being a coherent state, coherence can keeps long times when the initial average number is very small. In the second type of condition, e.g. the internal state being a superposition and motional state being a vacuum, a concrete form of coherent state for center-of-mass mode is obtained in another case of very small phonons. These properties for two trapped ions are useful for quantum computation. Because quantum manipulation involves at lest two-bit operations, and only we know the dynamical properties of the ions very clearly, can we manipulate them well and truly. In addition, the model for this system may be realized in present experimental conditions[14], which will result in directly the tests of the predictions from this paper.

This paper is organized as follows. In section II, we firstly deduce the JCM model for n trapped ions interacting with lasers in appropriate conditions, and then present its exact solution for the case of two trapped ions. In section III, we reveal the collapses and revivals for the occupation of internal states of the two trapped ions in two different types of initial condition. The coherence and squeezing for the vibratic motion of center-of-mass mode are discussed in section IV. Finally a summary and some conclusions are given in section V.

II. INTERACTION MODEL

For the sake of generality, we consider for a while n ions trapped in a linear trap which are strongly bounded in the y and z directions but weakly bounded in a harmonic potential in x direction. Assuming that the n ions are illuminated simultaneously with a dispersive beam

(or beams) resonant to the first red side-band of center-of-mass mode, so that the effects from the spectator motional modes can be neglected because of the very large off-resonant reason[15]. The Hamiltonian for this system is

$$H = H_0 + H_{int}, \quad (1)$$

$$H_0 = \nu(a^+a + 1/2) + \omega \sum_{i=1}^n \sigma_{iz}/2, \quad (2)$$

$$H_{int} = \sum_{i=1}^n \frac{\Omega}{2} \left\{ \sigma_{+i} e^{i\eta(a+a^+)} e^{-i(\omega-\nu)t} + H.C. \right\} \quad (3)$$

where ν and a^+ (a) are the frequency and ladder operators of the center-of-mass mode. ω is the energy difference between the ground state $|0\rangle$ and the long-lived metastable excited state $|1\rangle$ of each ion. For simplicity we further assume that the Rabi frequency and Lamb-Dicke parameter for each ion are same. If all ions are cooled under Lamb-Dicke limit, then we can expand the Hamiltonian up to the first order in η . Transforming the Hamiltonian (3) to the interaction picture with respect to H_0 and making use of rotating wave approximation, we have,

$$H_{int} = -i\Omega\eta(a^+J_- - aJ_+) \quad (4)$$

where $J_{\pm} = J_x \pm iJ_y$ with $J_{\alpha} = \sum_{i=1}^n \sigma_{i\alpha}$ ($\alpha = x, y, z$) being the three components of total spin operator of n ions.

E.q.(4) is namely the so-called JCM Hamiltonian for n ions interacting with a laser beam (or beams) in an ion trap. Compared with the traditional JCM Hamiltonian, the total spin ladder operators of n ions replaced the spin ladder operators of one ion. When any one of the n ions is excited, the quantum number of J_z component of total spin operator increases by one, and in the meantime, the quantum number of center-of-mass mode decreases by one. From this mode, we would naturally like to look forward to some similar properties as shown in standard JCM.

In general case, searching for the solution of E.q.(4) is an open question. Below we will only discuss the case that including only two trapped ions. In this case the last term in E.q.(2) can be rewritten as ωJ_z , and the quantum number corresponding to J_z component only have three values: 1, 0, -1 (the value “1” means both the two ions are excited, “0” means one of the two ions is excited, and “-1” means both the two ions are in ground state); Noting that in this system, the total excitations (including electron excitations and excitations of center-of-mass mode) is a conservation, thus we can arrange the eigenstates of Hamiltonian H_0 in the order of total excitations,

$$|1, n-2\rangle^n, |0, n-1\rangle^n, |-1, n\rangle^n; |1, n-1\rangle^{n+1}, |0, n\rangle^{n+1}, |-1, n+1\rangle^{n+1}; \dots \quad (5)$$

Where the first numbers in each Dirac symbols represent the quantum numbers of J_z component and the second ones represent the excitations of center-of-mass mode. The superscripts above each Dirac symbols represent the total excitations.

In the Hilbert space constituted by above basis, the interaction Hamiltonian (4) is a diagonal matrix made up of 3×3 sub-matrices. Every sub-matrix represents a subspace corresponding to a definite excitation. In the subspace including n excitations, we can get the exact propagator for Hamiltonian (4) after a tedious deduction,

$$U = \begin{bmatrix} \frac{1}{2n-1} [(n-1) \cos \beta + n] & -\sqrt{\frac{n-1}{2n-1}} \sin \beta & \frac{\sqrt{n(n-1)}}{2n-1} [-\cos \beta + 1] \\ \sqrt{\frac{n-1}{2n-1}} \sin \beta & \cos \beta & -\sqrt{\frac{n}{2n-1}} \sin \beta \\ \frac{\sqrt{n(n-1)}}{2n-1} [-\cos \beta + 1] & \sqrt{\frac{n}{2n-1}} \sin \beta & \frac{1}{2n-1} [n \cos \beta + (n-1)] \end{bmatrix} \quad (6)$$

where $\beta = \sqrt{2(2n-1)}\eta\Omega t$.

III. COLLAPSES AND REVIVALS

In order to study the phenomena of collapses and revivals of the two trapped ions, imitating traditional process, we assume that initially the internal states of the two ions

are both in the ground states and the external motional state of center-of-mass mode in a coherent state $|\alpha\rangle$. Thus the whole initial state of the system is,

$$\rho(0) = \sum_{m', m''=0}^{\infty} q(m') q^*(m'') | -1 \ m' \rangle \langle -1 \ m'' | \quad (7)$$

with $q(m) = \exp(-\frac{1}{2} |\alpha|^2) \frac{\alpha^m}{\sqrt{m!}}$ are the coefficients for a coherent state expanding in number state basis.

The density operator of the internal states can be obtained by tracing over the vibrational motion of center-of-mass mode. With above equations, we can get the diagonal elements of the density-matrix of internal states for this two-ion system,

$$\rho_{11}(t) = \sum_{n=1}^{\infty} \frac{n(n-1)}{(2n-1)^2} (-\cos \beta + 1)^2 p(n) \quad (8)$$

$$\rho_{00}(t) = \sum_{n=1}^{\infty} \frac{n}{2n-1} (\sin \beta)^2 p(n) \quad (9)$$

$$\rho_{-1-1}(t) = p(0) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} (n \cos \beta + n - 1)^2 p(n) \quad (10)$$

with

$$p(n) = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!} \quad (11)$$

is the probability distribution of phonon number for the coherent state of center-of-mass mode.

According to E.q.(8)-(11), we plotted the time evolution of diagonal elements of density-matrix of the internal states as Fig.1, which describes the time-evolution of the occupation-probability of electronic energy-levels. We choose the initial average phonon number $|\alpha|^2 = 8$, and the typical parameters $\eta = 0.1$ and $\Omega = 2\pi \times 500 \text{ kHz}$ (These typical parameters are used throughout this paper, below we will no longer narrate them repeatedly). From these curves, we can see clearly collapses and revivals for the electronic state-occupation of the two trapped ions. It is worthwhile to point out that, as in the traditional standard JCM, the

effect of collapses and revivals in here also increases with the glowing of the average phonon number. However the average phonon number may not be very large in here, because we assumed that the ions are cooled to be under the Lamb-Dick limit. In order to observe collapses and revivals in the case of large number of average phonons, we must look for the solution of the system far from Lamb-Dick limit. This is a problem awaiting to be solved.

Another possibility to obtain collapses and revivals for this system is letting the initial state of the ions in a superposition and the vibratic motion in a thermal state or vacuum state. This seems to be a more reasonable assumption, because the vibratic motion of the ions seems more to incline to a thermal state in general. For simplicity, we will suppose the motional state being in a vacuum, because in present conditions we can almost cool the ions to their vibratic ground state. Thus the whole initial state of this system is,

$$|\Psi(0)\rangle = (a|1\rangle + be^{i\varphi_1}|0\rangle + ce^{i\varphi_2}|-1\rangle) \otimes |0\rangle_{vib} \quad (12)$$

with $a^2 + b^2 + c^2 = 1$.

Similar to the deduction for coherent initial state, we have the diagonal elements of the density matrix for this kind of initial condition,

$$\rho_{11} = \frac{a^2}{9}[2 + \cos(\sqrt{3}\xi t)]^2 \quad (13)$$

$$\rho_{00} = \frac{a^2}{3}\sin^2(\sqrt{3}\xi t) + b^2\cos^2(\xi t) \quad (14)$$

$$\rho_{-1-1} = \frac{2}{9}a^2[1 - \cos(\sqrt{3}\xi t)]^2 + b^2\sin^2(\xi t) + c^2 \quad (15)$$

with $\xi = \sqrt{2}\eta\Omega$. Numerical calculation indicated that only ρ_{-1-1} reveals clear collapse and revival. However, we may expect more clear collapses and revivals when the number of superposition-ion increased. Because the degree of coherence between atoms will increase with the addition of ions. The picture for this case is presented in fig.2.

In addition, we studied, by way of parenthesis, the behaviors of internal state of the two ions for initial conditions that the two ions being ground and the vibratic motion being in

number, thermal and squeezing states. Some results displayed in fig.3. The amplitude of Rabi oscillation for the occupation of internal state $|\uparrow\uparrow\rangle$ is almost between 0 and 1 when the motional state be in a number state. But when the motional state is in a thermal or a squeezing state, the occupation-probability of $|\uparrow\uparrow\rangle$ is much smaller.

IV. COHERENCE AND SQUEEZING

In quantum optics, coherence and squeezing are two important aspects to describe properties of a radiative field. Coherence implies, in some sense, the interference between different photons. And squeezing is a phase-dependent reduction of the noise of the electric field strength (or, of one of the field quadratures) below the vacuum level[16]. Now we borrow these ideas to describe the properties of vibratic modes of trapped ions. In this case, they represent the coherence between phonons and the squeezing of position or momentum of COM mode respectively.

A quantitative measure for coherence can be defined by a fraction of the number of coherent phonons,

$$\langle n(t) \rangle_{coh} = |\langle a \rangle|^2 = \left| \sum_{n=0}^{\infty} \sqrt{n+1} \rho_{n(n+1)}(t) \right|^2 \quad (16)$$

to the total phonon number,

$$\langle n(t) \rangle = \sum_{n=0}^{\infty} n \rho_{nn}(t). \quad (17)$$

As for squeezing, we define the two quadratures of COM motion as,

$$\begin{aligned} x &= g(a + a^+) \\ p &= ig(a - a^+) \end{aligned} \quad (18)$$

where g may be regarded as the structure constant of a harmonic oscillator. A quantitative condition for squeezing is that the normally ordered variance is under zero:

$$\langle : (\Delta O)^2 : \rangle < 0 \quad (19)$$

where O means x or p in E.q.(18). The normally ordered variance for x and p can be written as,

$$\begin{aligned} \langle : (\Delta x)^2 : \rangle &= 2g^2 \langle n(t) \rangle + 2g^2 \left\{ \sum_{n=0}^{\infty} \sqrt{(n+1)(n+2)} R_e[\rho_{n(n+2)}(t)] \right\} \\ &\quad - 4g^2 \left\{ \sum_{n=0}^{\infty} \sqrt{(n+1)} R_e[\rho_{n(n+1)}(t)] \right\}^2 \end{aligned} \quad (20)$$

$$\begin{aligned} \langle : (\Delta p)^2 : \rangle &= 2g^2 \langle n(t) \rangle - 2g^2 \left\{ \sum_{n=0}^{\infty} \sqrt{(n+1)(n+2)} R_e[\rho_{n(n+2)}(t)] \right\} \\ &\quad - 4g^2 \left\{ \sum_{n=0}^{\infty} \sqrt{(n+1)} I_e[\rho_{n(n+1)}(t)] \right\}^2 \end{aligned} \quad (21)$$

From above two equations, we can see that in order to obtain squeezing, the phonon density-matrix must exhibit nonvanishing off-diagonal elements $\rho_{n(n+1)}$ and /or $\rho_{n(n+2)}$. There may exist some different possibilities to generate them, e.g. initially (1) internal states be in a mono-state and vibratic motion be in a coherent state (or other states with coherence, say, binomial state[17].); (2) motional state be in a thermal or vacuum (means no coherence), but internal state be in coherent superposition; (3) internal state be in a superposition and, in the same time, vibratic motion be in a coherent state. Below we will still analysis the two different types of initial condition discussed in section III.

For the first case, e.g. initially the internal states of the two ions are both in ground and motional state of COM mode is in a coherent state. After a complicate deduction, we get the density operator of vibratic motion,

$$\begin{aligned} \rho_{lm}(t) &= \frac{[(l+1)(l+2)(m+1)(m+2)]^{1/2}}{(2l+3)(2m+3)} \left[1 - \cos(\sqrt{2l+3}\xi t) \right] \times \\ &\quad \left[1 - \cos(\sqrt{2m+3}\xi t) \right] q(l+2)q^*(m+2) + \left[\frac{(l+1)(m+1)}{(2l+1)(2m+1)} \right]^{1/2} \times \\ &\quad \sin(\sqrt{2l+1}\xi t) \sin(\sqrt{2m+1}\xi t) q(l+1)q^*(m+1) + [(2l-1)(2m-1)]^{-1} \times \\ &\quad \left[l \cos(\sqrt{2l-1}\xi t) + l-1 \right] \left[m \cos(\sqrt{2m-1}\xi t) + m-1 \right] q(l)q^*(m) \end{aligned} \quad (22)$$

where $q(m)$ is shown in E.q.(7).

It is easily imaginable that the average phonon number $\langle n(t) \rangle$ doesn't depend on φ , but increases with the augmentation of initial average number r^2 or $\langle n(0) \rangle$. Given a definite initial phonon number r^2 , $\langle n(t) \rangle$ reveals also obvious collapses and revivals as time passes (see Fig.4). Moreover, this effects also become clear with the increasing of the initial phonon number. In fact, the collapses and revivals of phonon number relate to the collapses and revivals of the occupation of internal states. During the time that the number of phonons collapses, its number almost doesn't change. So the occupation of internal states also doesn't change, and collapses occur at this time. In order to compare with the collapses and revivals of the occupation of internal states in section III, we choose the same initial condition and parameters in Fig.4 as in Fig.1.

Coherence $\gamma = |\langle a \rangle|^2 / \langle n(t) \rangle$ is also independent on φ , but rely on initial phonon number $\langle n(0) \rangle$ and time t . Coherence γ oscillates damply on time and reduces rapidly with the increasing of initial phonon number(see Fig.5). Thus if we want to keep the coherence of vibratic motion for some long times, the initial average phonons must be very small.

The squeezing of position and momentum depends on $\langle n(0) \rangle$, φ and time t simultaneously.. But from an inspection of (20)~(22), we can first easily make sure that the minimums of $\langle (\Delta x)^2 \rangle$ and $\langle (\Delta p)^2 \rangle$ with respect to φ occur at $\varphi = 0$. Then by use of numerical calculation, we find that $\langle (\Delta p)^2 \rangle$ increases rapidly with the growing of initial average number (see Fig.6). When $\langle n(0) \rangle \geq 2.0$, the momentum-quadrature doesn't exist squeezing. The minimum of $\langle (\Delta p)^2 \rangle$ with respect to initial average number occurs at about $\langle n(0) \rangle = 0.51$. Given φ and $\langle n(0) \rangle$, $\langle (\Delta p)^2 \rangle$ is an oscillating function on time. In order to see squeezing clearly, we plotted the time-evolution of $\langle (\Delta p)^2 \rangle$ in Fig.7. As a comparison, we also plotted the time-evolution of $\langle (\Delta x)^2 \rangle$ in the same initial condition. The maximum squeezing of momentum-quadrature in this type of initial condition is about $\langle (\Delta p)^2 \rangle = -0.424g^2$ which occurs at about $\varphi = 0$, $\langle n(0) \rangle = 0.51$ and $t = 343\mu s$. Similarly, we can determine the minimum of $\langle (\Delta x)^2 \rangle$ and its location.

Next, we consider the case that the initial state of the system is Eq.(12). In this case we

can deduce the density operator of motional state only having the following nonvanishing elements,

$$\rho_{00}(t) = \frac{1}{9}a^2 \left[2 + \cos \left(\sqrt{3}\xi t \right) \right]^2 + b^2 \cos^2(\xi t) + c^2 \quad (23)$$

$$\rho_{11}(t) = \frac{1}{3}a^2 \sin^2 \left(\sqrt{3}\xi t \right) + b^2 \sin^2(\xi t) \quad (24)$$

$$\rho_{22}(t) = \frac{2}{9}a^2 \left[1 - \cos \left(\sqrt{3}\xi t \right) \right]^2 \quad (25)$$

$$\rho_{01}(t) = \rho_{10}^*(t) = \frac{1}{\sqrt{3}}ab \sin \left(\sqrt{3}\xi t \right) \cos(\xi t) e^{i\varphi_1} + bc \sin(\xi t) e^{i(\varphi_2 - \varphi_1)} \quad (26)$$

$$\rho_{02}(t) = \rho_{20}^*(t) = \frac{\sqrt{2}}{3}ac \left[1 - \cos \left(\sqrt{3}\xi t \right) \right] e^{i\varphi_2} \quad (27)$$

$$\rho_{12}(t) = \rho_{21}^*(t) = \frac{\sqrt{2}}{3}ab \left[1 - \cos \left(\sqrt{3}\xi t \right) \right] \sin(\xi t) e^{i\varphi_1}. \quad (28)$$

Similar to the above discuss, we can find average phonon number $\langle n(t) \rangle$ doesn't depend on φ_1 and φ_2 , and increases with the add of a , b , and also occurs collapse and revival on time.

The coherence γ reaches its maximum when $\cos(2\varphi_1 - \varphi_2) = 1$. And this maximum increases with the enhance of c . In the limits that $a = 0$, $b \rightarrow 0$ (but $b \neq 0$) and $c \rightarrow 1$, coherence $\gamma \rightarrow 1$. This implies that in the case of very small phonons(“weak field”), the vibratic motion approaches to a coherent state. In fact, in the above limits, the density-matrix elements (23)~(28) can be written as,

$$\rho_{00}(t) = 1 - \langle n(t) \rangle \quad (29)$$

$$\rho_{11}(t) = \langle n(t) \rangle \quad (30)$$

$$\rho_{01}(t) = \rho_{10}^*(t) = e^{i(\varphi_2 - \varphi_1)} \langle n(t) \rangle^{1/2} \quad (31)$$

and all other elements vanished completely (Noting $\langle n(t) \rangle = b^2 \sin^2(\xi t)$). These are just the density-matrix elements of coherent state up to the terms of the order $|\alpha|$ and $|\alpha|^2$ (see the second reference of [8]).

An inspection of (20)~(21) and (23)~(28), in combination with numerical calculation suggests that for any φ_1 , when $\varphi_2 = 0$, $b = 0$ and $c = 0.96$, the squeezing of momentum-quadrature reaches its maximum.. This maximum squeezing oscillate with a nearly equi-amplitude on time. When t satisfying $\cos(\sqrt{3}\xi t) = -1$, the maximum squeezing $\langle (\Delta p)^2 \rangle = -0.438g^2$ for momentum-quadrature has been obtained (see Fig.8). In fact, in the condition that $b = 0$, $a \rightarrow 0$, $c \rightarrow 1$, and $\cos(\sqrt{3}\xi t) = -1$, the nonvanishing density-matrix elements in (23)~(28) are,

$$\rho_{00}(t) = 1 - \frac{1}{2} \langle n(t) \rangle \quad (32)$$

$$\rho_{22}(t) = \frac{1}{2} \langle n(t) \rangle \quad (33)$$

$$\rho_{02}(t) = \rho_{20}^*(t) = \frac{1}{\sqrt{2}} e^{i\varphi_2} \sqrt{\langle n(t) \rangle} \quad (34)$$

where $\langle n(t) \rangle = 16a^2/9$ with t determined by $\cos(\sqrt{3}\xi t) = -1$. These are just the density-matrix elements of squeezing vacuum state (see the second reference of [8]). Thus we also obtained a “weak-field” squeezing state in another group of initial parameters. In the case that $\cos(\sqrt{3}\xi t) \neq -1$, the momentum quadrature also has squeezing but not reaches its maximum (see Fig 7).

It is worthwhile to point out that the coherence and squeezing discussed above includes the rapid time dependence, which, of course, is meaningless when slowly varying quantities are of interest. But we can slowen the frequency of time-dependence by decreasing Rabi frequency Ω . Moreover, in current experimental conditions, dealing with problems in microsecond is completely feasible.

V. SUMMARY AND CONCLUSION

We have studied the dynamical behaviors of two trapped ions interacting with lasers resonant to the first red side-band of center-of-mass mode. In small Lamb-Dick parameters and under rotating-wave approximation, the dynamics of the system can be described by JCM-like mode. An exact analytic solution for this type of JCM was presented. The results indicated obvious collapses and revivals for the occupation of internal states in two different types of initial condition. These collapses and revivals become more visible when the average phonon number increases.

Coherence and squeezing for vibratic motion of center-of-mass mode were also discussed in these two different types of initial condition. For one type of initial condition that the internal states being grounds and vibratic motion being in a coherent state, coherence can maintain long times when the initial average phonon number is very small. A maximum squeezing 42.4% for momentum-quadrature was obtained in appropriate parameters. For another type of initial condition that the internal state being in a superposition and the vibratic motion being vacuum, apparent expressions for coherent state and squeezing state were acquired in suitable conditions. The maximum squeezing for momentum-quadrature in this case is 43.8%.

We expect the results in this paper can bring some useful helps to other problems, especially the quantum computation or quantum information processing. Because the research for the dynamical properties of two trapped ions locate completely in the field of trapped-ion quantum computation. We also expect this paper can give rise to some other interesting discuss for system of multiple ions, such as squeezing and coherence of vibrational state, collapses and revivals in large Lamb-Dick parameters or for systems containing more than two ions.

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Fig.1. The occupations $\rho_{11}(t)$, $\rho_{00}(t)$, $\rho_{-1-1}(t)$ of the internal state are given as functions of time. Initially the motional state of COM mode be in a coherent state with average phonon number $|\alpha|^2 = 8$ and the two ions be in ground state.

Fig.2. The time evolution of $\rho_{-1-1}(t)$ for initially the internal state be in a superposition of E.q.(12) with $a = b = c = 1/\sqrt{3}$ and motional state be in a vacuum.

Fig.3. The time evolution of $\rho_{11}(t)$ for initially the internal states of the two ions are both in grounds and motional states are in number, thermal and squeezing state. In three cases the average phonon number $\langle n(0) \rangle = 3$. (a) number state, (b) thermal state, (c) squeezing state.

Fig.4. The average phonon number $\langle n(t) \rangle$ is shown as a function of time for the same initial conditions and parameters as in Fig.1.

Fig.5. The coherence γ is shown as a function of time and initial coherent phonon number for the same initial conditions and parameters as in Fig.1.

Fig.6. The evolution of $\langle (\Delta p)^2 \rangle$ as a function of the initial average number and time in given $\varphi = 0$.

Fig.7. $\langle (\Delta x)^2 \rangle$ (dot line), $\langle (\Delta p)^2 \rangle$ (full line) are shown as a function of time for initial parameters $\langle n(0) \rangle = 0.51, \varphi = 0$. When $t = 343\mu s$, momentum quadrature has a maximum squeezing $\langle (\Delta p)^2 \rangle = -0.424g^2$.

Fig.8. (a). $\langle (\Delta x)^2 \rangle$ (dot line), $\langle (\Delta p)^2 \rangle$ (full line) are shown as a function of time for initial parameters $c = 0.96, b = 0$. When t satisfies $\cos(\sqrt{3}\xi t) = -1$, momentum

quadrature has a maximum squeezing $\langle (\Delta p)^2 \rangle = -0.438g^2$.